

FIRST PRE-BOARD EXAMINATION:2020-21

CLASS : XII

SUBJECT:-MATHEMATICS

Max Marks:-80

Time: - 3 hours

General instructions

- (i) This question paper consists of two parts A and B. Each part is compulsory. Part A carries 24 marks and part B carries 56 marks.
 - (ii) Part A has objective type questions and part B has descriptive type questions.
 - (iii) There is no overall choice. However, an internal choice has been provided in both the parts A and B.
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PART – A :

1. This part consists of two sections- I and II.
 2. Section I comprises of 16 very short answer type questions.
 3. Section II contains 2 case studies. Each case study comprises of 5 case based MCQs. An examinee is to attempt any 4 out of 5 MCQs.
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PART- B :

1. This part consists of three sections- III, IV and V.
 2. Section III contains 10 questions of 2 marks each.
 3. Section IV contains 7 questions of 3 marks each.
 4. Section V contains 3 questions of 5 marks each
 5. Internal choice is provided in 3 questions of section- III, 2 questions of section-IV and 3 questions of section –V. You have to attempt only one of the alternatives in all such questions.
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PART A
SECTION-I

1. Let $A=\{1,2,3,4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$.
2. State the reason why the relation $R = \{(a, b) : a < b^3\}$ on the set R of real

number is not reflexive.

3. Find the value of $\sin^{-1}[\cos(43\pi/5)]$.

OR

Write the range of one branch of $\sin^{-1}x$, other than the principal value branch.

4. Let R is a relation in the set $A=\{1,2,3\}$ given by $R=\{(1,2),(2,1)\}$, write the pair of element(s) to add so that relation become transitive.

OR

Check whether the function $f:R \rightarrow R$ given by $f(x)=\cos x$ is one –one or not.

5. Find the value of A^2 , where A is a 2x2 matrix whose elements are given by

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

OR

Given that A is a square matrix of order 3x3 and $|A| = -5$. Find $|\text{adj}A|$.

6. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ then find the value of $a-3b$.

7. If A and B are square matrices of order 3 each, $|A|=2$ and $|B|=3$. Find the value of $|3AB|$.

8. Find the sum of the order and the degree of differential equation $\frac{d^2y}{dx^2} +$

$$\sqrt[3]{\frac{dy}{dx}} + (5+x) = 0.$$

9. Evaluate $\int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$

OR

Evaluate $\int_e^{e^2} \frac{dx}{x \log x}$

10. Write the vector equation of line through the point (5,2,-7) and which is parallel to the vector $3\hat{i} + 4\hat{j} - 11\hat{k}$.

11. Find the intercept cut off by the plane $x-y+5z=7$ on x and z axis.

12. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of $\vec{a} + \vec{b}$.

13. Find the area bounded by $y=x^2$, the x axis and the lines $x=-1$ and $x=1$.

14. Find the direction cosines of the normal to yz plane?

15. Evaluate: $P(A \cup B)$; if $2P(A)=P(B)=5/13$ and $P(A|B)=2/5$.

16. Let A and B be events with $P(A)=3/5$, $P(B)=3/10$ and $P(A \cap B)=1/5$. Are A and B are independent? Justify.

Section II

Both the Case Study based questions are compulsory. Attempt any 4 sub parts from each question. (Each question carries 1 mark).

17. An insurance company insure three type of vehicles *i.e.*, type A, B and C. If it insured 12000 vehicles of type A, 16000 vehicles of type B and 20,000 vehicles of type C. Survey report says that the chances of their accident are 0.01, 0.03 and 0.04 respectively.

Based on the informations given above, write the answer of following:

- (i) The probability of insured vehicle of type C is
(a) $\frac{5}{12}$ (b) $\frac{4}{12}$ (c) $\frac{7}{12}$ (d) $\frac{3}{12}$
- (ii) Let E be the event that insured vehicle meets with an accident then $P\left(\frac{E}{A}\right)$ is
(a) 0.09 (b) 0.01 (c) 0.07 (d) 0.06
- (iii) Let E be the event that insured vehicle meets with an accident then $P(E)$ is
(a) $\frac{38}{1200}$ (b) $\frac{32}{1200}$ (c) $\frac{24}{1200}$ (d) $\frac{35}{1200}$
- (iv) The probability of an accident that one of the insured vehicle meets with an accident and it is a type C vehicle
(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{5}{7}$ (d) $\frac{4}{7}$
- (v) One of the insured vehicles meets with an accident and it is not of type A and C.
(a) $\frac{12}{35}$ (b) $\frac{20}{35}$ (c) $\frac{1}{35}$ (d) $\frac{17}{35}$
18. In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meter of water.
Using above informations answer the following:
- (i) If the edge of square is x meter and height of tank is y m then correct relation is
(a) $x^2y=500$ (b) $xy=500$ (c) $x^2y^2=500$ (d) $x^2 + y =500$
- (ii) Relation for surface area of tank in terms of x and y is
(a) $2x^2 + 4xy$ (b) $x^2 + xy$ (c) $2x^2 + 2xy$ (d) $x^2 + 4xy$
- (iii) The surface area of tank is minimum when x is equal to
(a) 8 m (b) 20m (c) 10m (d) 5m
- (iv) The minimum surface area of tank is
(a) 200 sq m (b) 300 sq m (c) 250 sq m (d) 400 sq m
- (v) If size of square base of tank become twice and height remains same, then the volume

of tank will increase by cubic meters *i.e.*,

- (a) 500 cu m (b) 1000 cu m (c) 1500 cu m (d) 2000 cu m

Part B

Section III (Two marks questions)

19. Express $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$.
20. If A is square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1} .

21. Consider $f: R_+ \rightarrow [4, \infty)$ given that $f(x) = x^2 + 4$. Show that f is one-one and onto.
22. Find the value(s) of k so that the following function is continuous at $x = \pi/2$

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \pi/2 \\ 2, & \text{if } x = \pi/2 \end{cases}$$

23. Find equation of tangent to the curve $y = \cos x$ at $x = \frac{\pi}{3}$.

24. Evaluate $\int \frac{dx}{x^2(x+1)}$

OR

Find $\int \frac{dx}{e^x + e^{-x}}$.

25. Find the area of the region bounded by $y^2 = 8x$ and the line $x = 3$.
26. Solve the differential equation: $\frac{dy}{dx} = -4xy^2$ given that $y(0) = 1$.
27. A black and a red dice are rolled together. Find the conditional probability of obtaining the sum 8, given that the red dice resulted in a number less than 4.

OR

Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$ Find $P(\bar{E} | \bar{F})$.

28. Find the image of point (1, 0, 0) on the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Section IV (Three marks questions)

29. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x .

OR

If $x = a \sec t$, $y = b \tan t$ find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{6}$.

30. Find the intervals in which the function f given by $f(x)=(x+1)^3(x-3)^3$ is strictly increasing or decreasing.

31. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

32. Find the area of the region bounded by the curves $x^2+y^2=4$, $y=\sqrt{3}x$ and x axis in the first quadrant.

OR

Find the area of the curve $9x^2+y^2=36$ using integration.

33. Solve $\frac{dy}{dx} = \frac{y}{x} - \cot \frac{y}{x} \cdot \cos \frac{y}{x}$

34. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

35. Integrate the function $\frac{x+2}{\sqrt{x^2+2x+3}}$

Section V(Five marks questions)

36. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$, $2y - 3z = 1$ and $3x - 2y + 4z = 2$

OR

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations $x+2y+z=4$, $-x + y + z=0$ and $x-3y+z=2$.

37. Find the maximum and minimum values of $5x + 2y$ subject to constraints $2x + 3y \geq 6$, $x - 3y \leq 3$, $3x + 4y \leq 24$, $x \geq 0$ and $y \geq 0$.

38. Find the equation of a line passing through the point $(2,1,3)$ and perpendicular to the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

OR

Find the equation of the plane passing through the point $(0, 1, 0)$ and $(3, 4, 1)$ and parallel to the line $\frac{x+3}{2} = \frac{y-3}{2} = \frac{z-2}{5}$.